Ex: Prove that $\sum_{i=1}^{n} \frac{1}{\sqrt{i}} > \sqrt{n}$ for all integers $n \geq 2$

Inductive Step: $\sum_{i=1}^{k} \frac{1}{\sqrt{k}} > \sqrt{k}$ let $k \geq 2$ for some integers k.

I.H.

We want to prove $\sum_{i=1}^{k+1} \frac{1}{\sqrt{i}} > \sqrt{k+1}$

now, $\sum_{i=1}^{k+1} \frac{1}{\sqrt{i}} = \sum_{i=1}^{k} \frac{1}{\sqrt{i}} \left(\frac{1}{\sqrt{k+1}}\right) > \sqrt{k} + \frac{1}{\sqrt{k+1}}$

$$= \frac{\sqrt{k} \cdot \sqrt{k+1} + 1}{\sqrt{k+1}}$$

$$> \frac{\sqrt{k}\sqrt{k+1}}{\sqrt{k+1}}$$

$$= \frac{k+1}{\sqrt{k+1}}$$

$$= \sqrt{k+1}$$

thus $\sum_{i=1}^{n} \frac{1}{\sqrt{i}} > \sqrt{n}$ for all integers $n \geq 2$

This means $\int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx = \infty$

Ex Prove that $\sum_{i=n}^{2n-1} (2i+1) = 3n^2$ for all integers $n \geq 1$

Base case $n=1$

$$\sum_{i=1}^{1} (2(1)+1) = 3 = 3(1) = 3(1)^2$$

Inductive Step let $k \geq 1$ for some integer k

I.H.

suppose $\sum_{i=k}^{2k-1} (2i+1) = 3k^2$

Prove $\sum_{i=k+1}^{2(k+1)-1} (2i+1) = 3(k+1)^2$

$$\sum_{i=k+1}^{2k+1} (2i+1) = \left(\sum_{i=k}^{2k-1} (2i+1)\right) - (2k+1) + (2(2k)+1) + (2(2k+1)+1)$$